Handout No 2
Course 2.05

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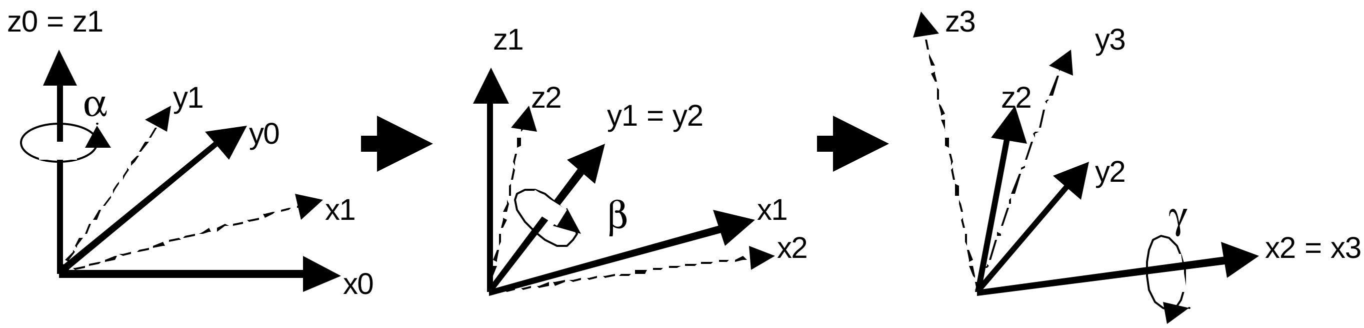
Euler ZYX Convention



Rotation about z0 of angle α + Rotation about y1 of angle β + Rotation about x2 of angle γ

$$T_{0,3} = T_{0,1}T_{1,2}T_{2,3} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} \cos(a) & -\sin(a) & 0 \\ \sin(a) & \cos(a) & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos(b) & 0 & \sin(b) \\ 0 & 1 & 0 \\ -\sin(b) & 0 & \cos(b) \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(g) & -\sin(g) \\ 0 & \sin(g) & \cos(g) \end{bmatrix} =$$

$$\begin{bmatrix} \cos(a)\cos(b) & \cos(a)\sin(b)\sin(g)-\sin(a)\cos(g) & \cos(a)\sin(b)\cos(g)+\sin(a)\sin(g) \\ \sin(a)\cos(b) & \sin(a)\sin(b)\sin(g)+\cos(a)\cos(g) & \sin(a)\sin(b)\cos(g)-\cos(a)\sin(g) \\ -\sin(b) & \cos(b)\sin(g) & \cos(b)\cos(g) \end{bmatrix}$$

Computation of Euler ZYX angles:

If $(r_{11} = r_{21} = 0 \Leftrightarrow \cos(b) = 0)$, then $\begin{cases} b = \frac{p}{2}, \\ a = 0, \\ g = \tan_2^{-1}(r_{12}, r_{22}) \end{cases}$ Else, then $\begin{cases} b = \tan_2^{-1}(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2}) \\ a = \tan_2^{-1}(r_{21}, r_{11}) \\ g = \tan_2^{-1}(r_{32}, r_{33}) \end{cases}$

Roll Pitch Yaw (RPY) Convention

Rotation about x0 of angle γ + Rotation about y0 of angle β + Rotation about z0 of angle α

All rotations are about fixed frame (x0, y0, z0) base vectors

Homogeneous Matrix and Angles are identical between these two conventions:

Roll Pitch Yaw XYZ (γ,β,α) ⇔ Euler ZYX (α,β,γ)

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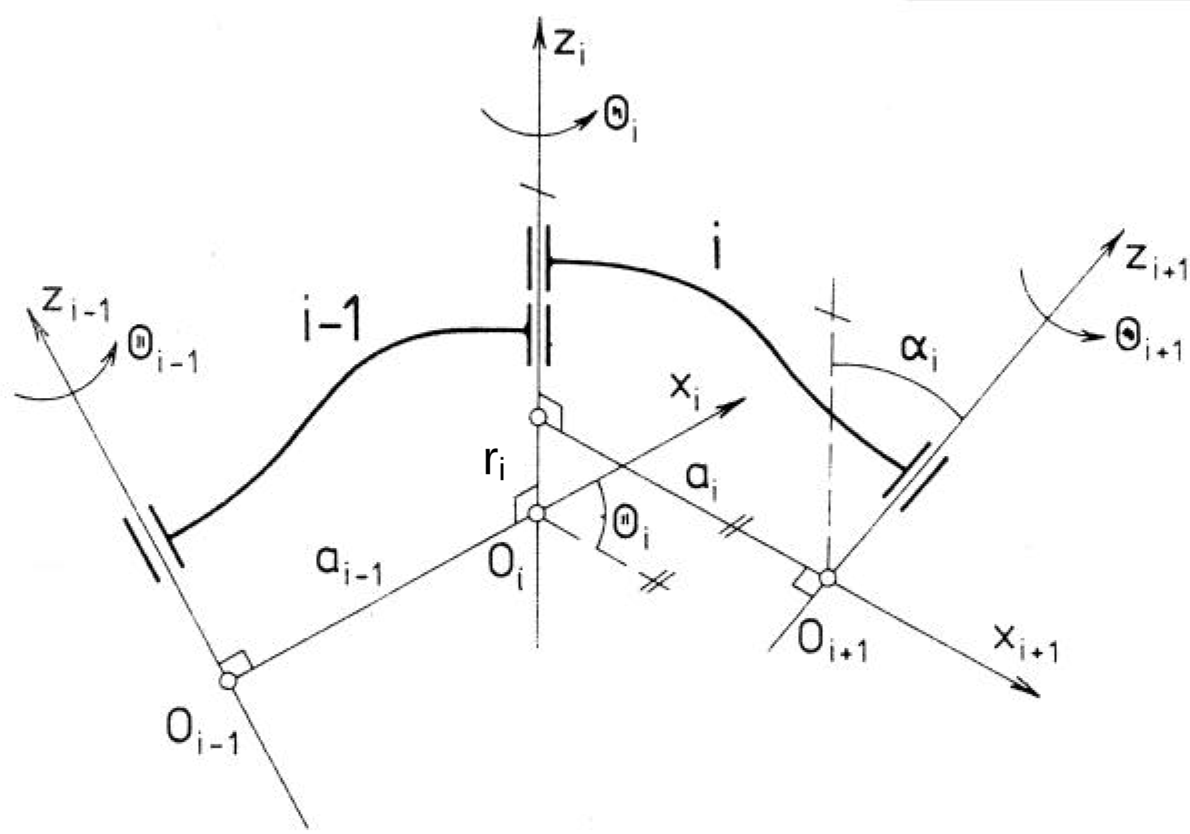
Denavit-Hartenberg Notation



Transformations of link L_i from frame (O_i, X_i, Y_i, Z_i) to frame $(O_{i+1}, X_{i+1}, Y_{i+1}, Z_{i+1})$:

Rotation (Z_i, θ_i) + Translation (Z_i, r_i) + Translation (X_{i+1}, a_i) + Rotation (X_{i+1}, α_i)

$$A_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & 0 \\ \sin(\theta_i) & \cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & r_i \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & \cos(\alpha_i) & -\sin(\alpha_i) & 0 \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} \cos(\theta)_i & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\ \cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & a_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & r_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Global Manipulator Transformation Matrix: $T_{1,n+1} = \prod_{i=1}^{i=n} A_i(\theta_i, r_i, a_i, \alpha_i)$